

# Improving tracking performance by learning from past data

Angela P. Schoellig

Doctoral examination – July 30, 2012

Advisor: Prof. Raffaello D'Andrea // Co-advisor: Prof. Andrew Alleyne



# Improving tracking performance by learning from past data = *experience*

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# MOTIVATION

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HUMANS learn from experience.

We learn from mistakes and get better through practice.



We constantly adapt to changing environments.



# MOTIVATION

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AUTOMATED SYSTEMS typically make the *same mistakes* over and over again when performing a task repeatedly.

Why?

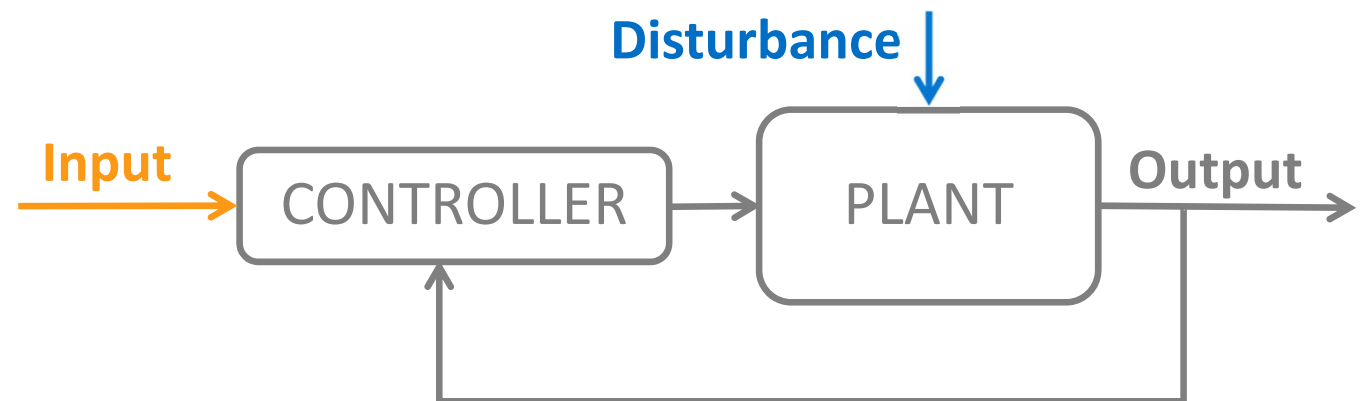
Robots of a car assembly line.



# MOTIVATION

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AUTOMATED SYSTEMS are typically operated using feedback control:

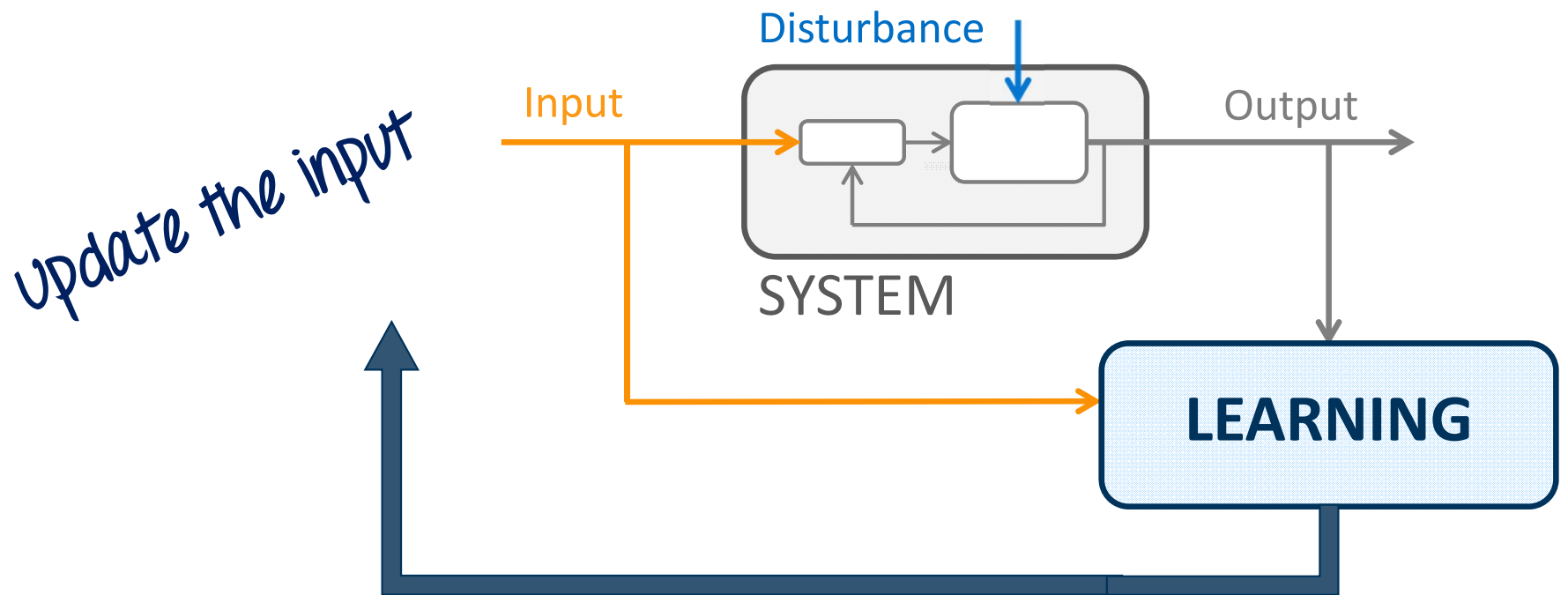


## Performance limitations:

- Causality of disturbance correction: “first detect error, then react”.
- Model-based controller design; model  $\neq$  real system.

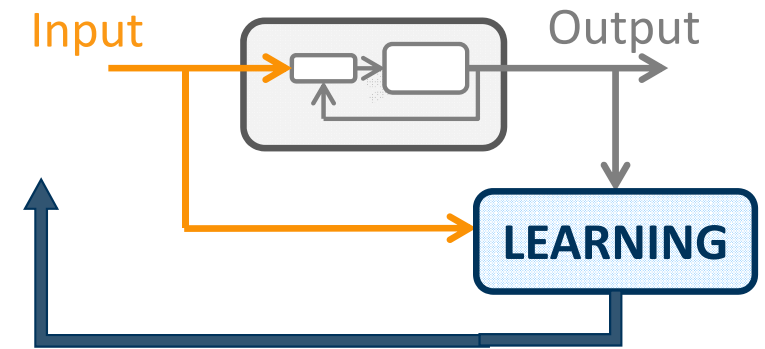
# GOAL

Improve the performance over causal, feedback control by learning from previous experiments.



# SCOPE OF WORK

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## Learning task:

Following a predefined trajectory.

## Approach:

- Model-based learning based on *a priori* knowledge of the system dynamics.
- Adaptation of the input.

## Potential:

Acausal action, anticipating repetitive disturbances.

# OVERVIEW

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## I. Introduction

- a. Testbed: The Flying Machine Arena
- b. Motivation for learning

II. **Project A.** Iterative learning for precise trajectory following:  
single-agent and multi-agent results. *FOCUS OF THIS TALK*

III. **Project B.** Learning of feed-forward parameters for rhythmic flight performances

IV. Summary



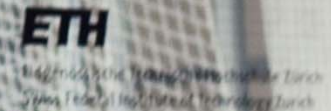
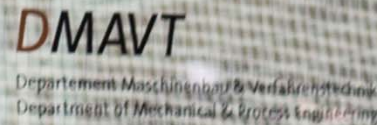
TESTBED, see [www.flyingmachinearena.org](http://www.flyingmachinearena.org)

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visitor platform →

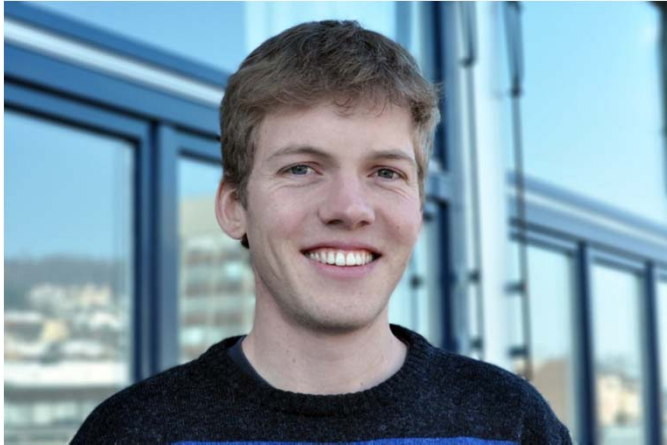
**www.FlyingMachineArena.org**

A space where flying robots live and learn.



# THE TEAM

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Mark Müller



Markus Hehn



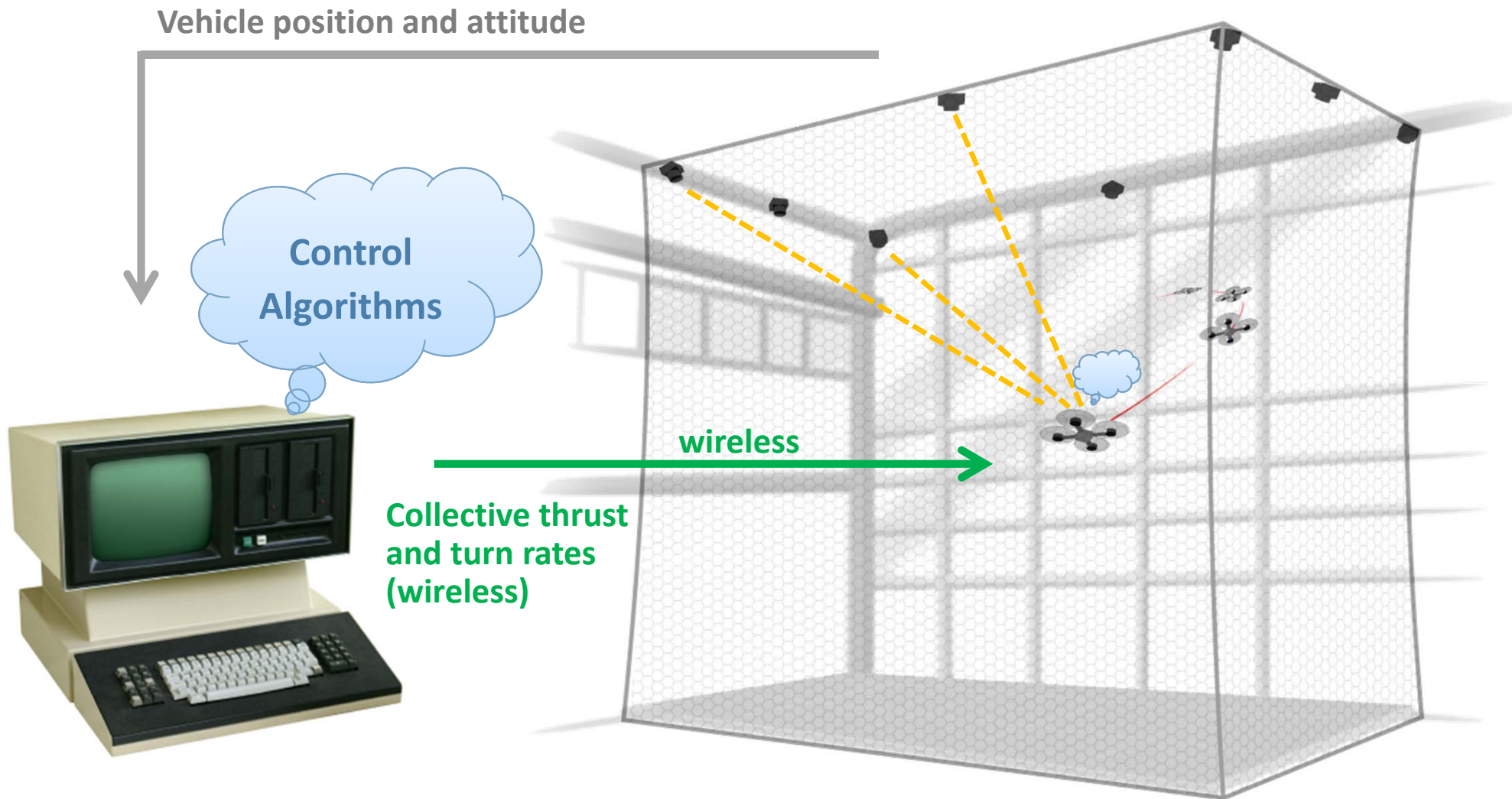
Sergei Lupashin



Federico  
Augugliaro

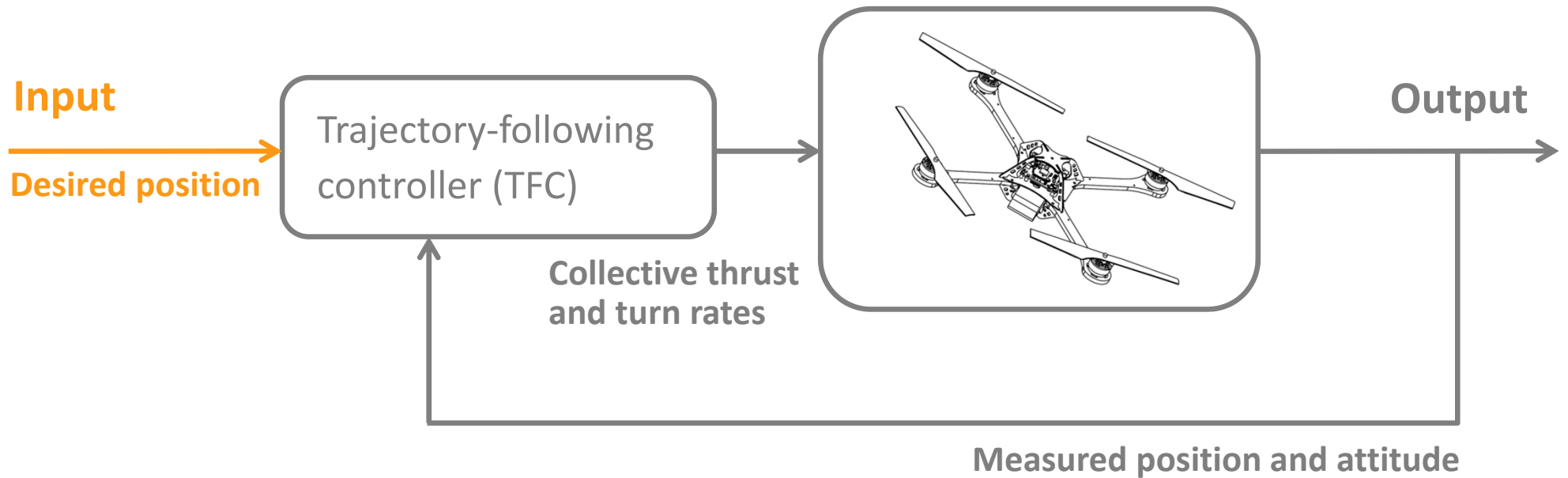


# THE FLYING MACHINE ARENA



# OPERATION

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# MOTIVATION: PROJECT A

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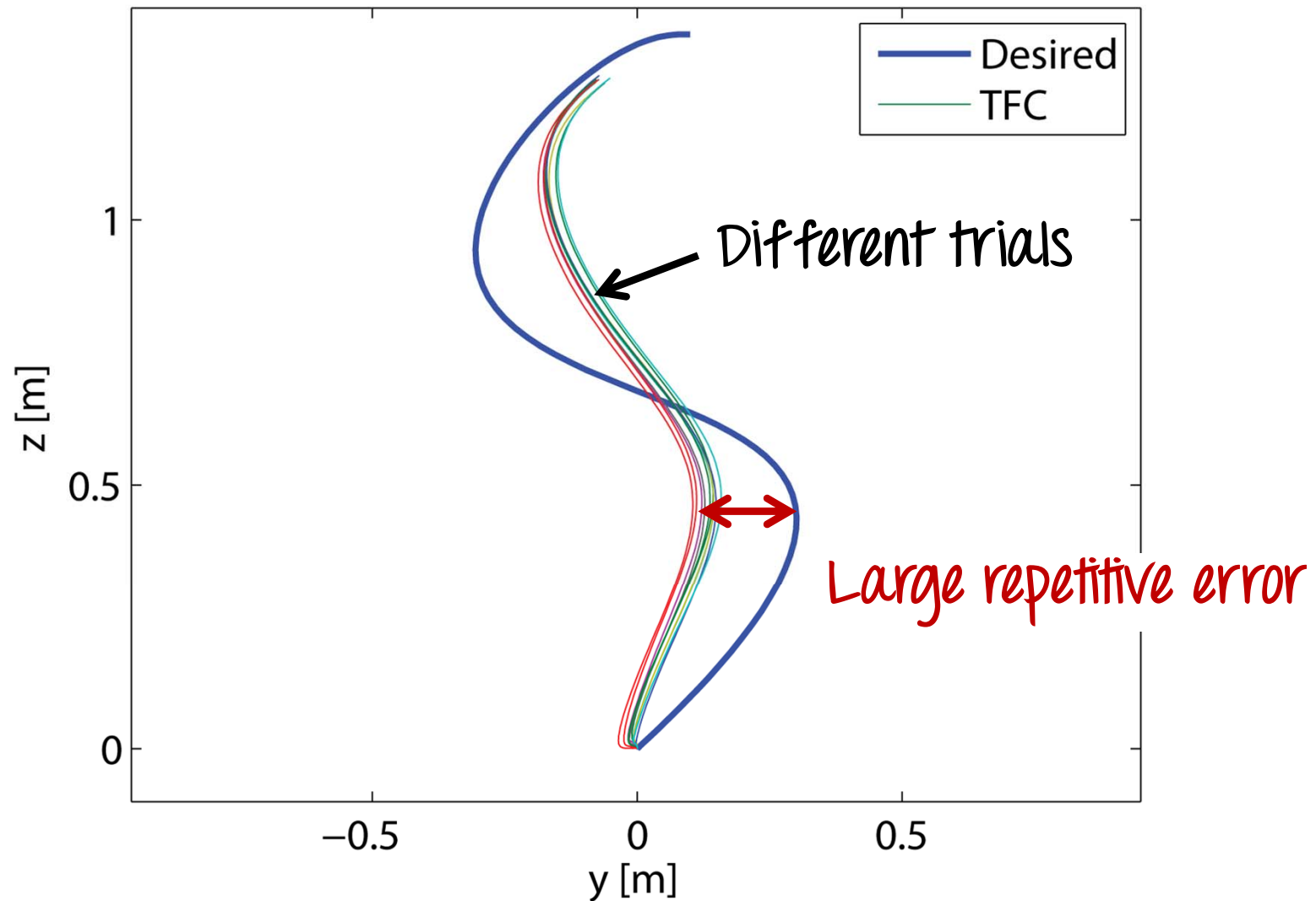
**Desired motion.**



# MOTIVATION: PROJECT A

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Performance with trajectory-following controller.



# OVERVIEW

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## I. Introduction

## II. Project A. Iterative learning for precise trajectory following

- a. Learning approach
- b. Results

## III. Project B. Learning of feed-forward parameters for rhythmic flight performances

## IV. Summary

# A | PUBLICATIONS

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## *Peer-reviewed publications*

Schoellig, A. P. and R. D'Andrea (2009):

“Optimization-based iterative learning control for trajectory tracking.” In *Proceedings of the European Control Conference (ECC)*.

Schoellig, A. P., F. L. Mueller, and R. D'Andrea (2012):

“Optimization-based iterative learning for precise quadcopter trajectory tracking.” *Autonomous Robots*.

Mueller, F.L., A. P. Schoellig, and R. D'Andrea (2012):

“Iterative learning of feed-forward corrections for high-performance tracking.” To appear in *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*.

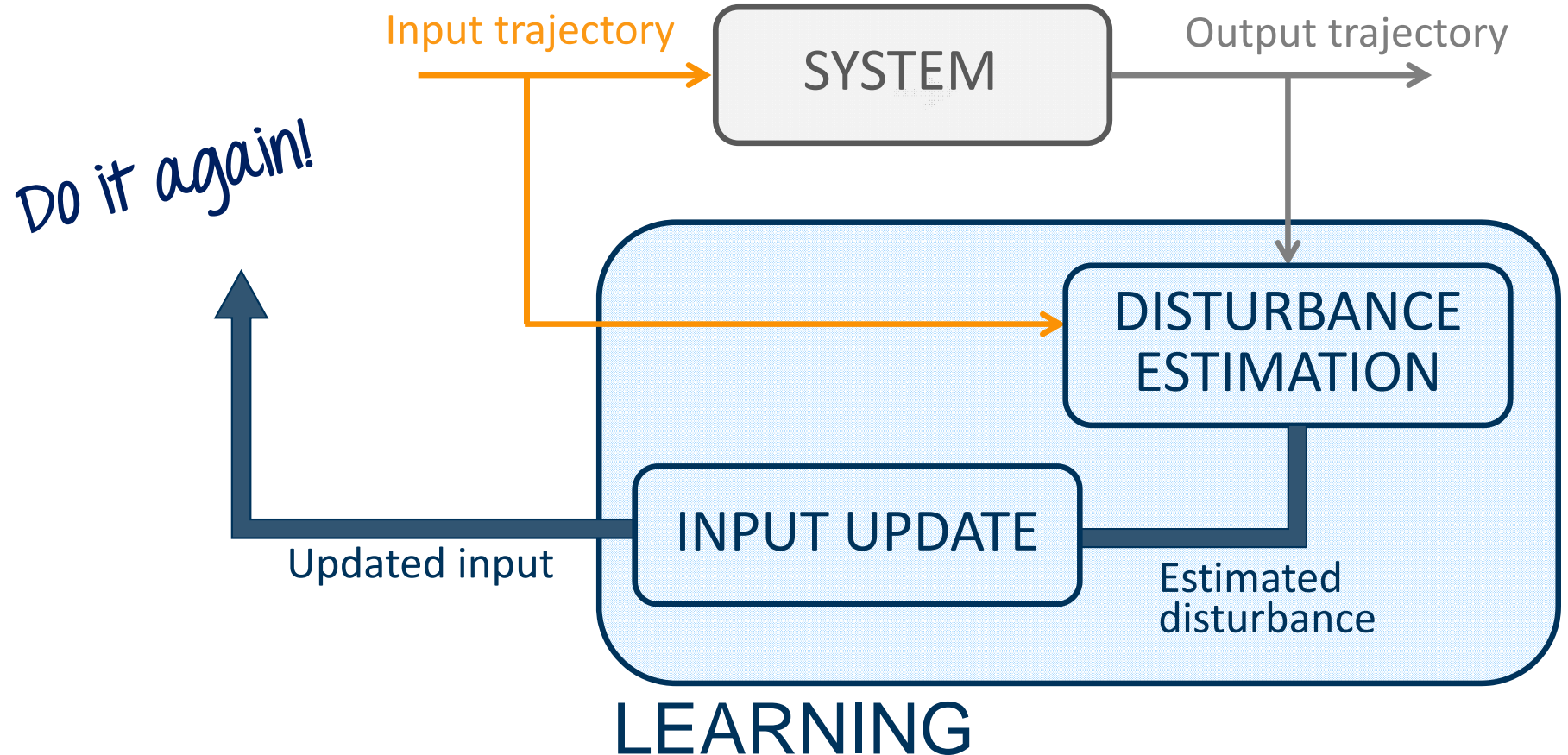
**Joint work with Fabian L. Mueller (Master student).**



# A | LEARNING APPROACH

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**Features:** Learning through a repeated operation, updating full input trajectory after each trial.



# A | LEARNING APPROACH

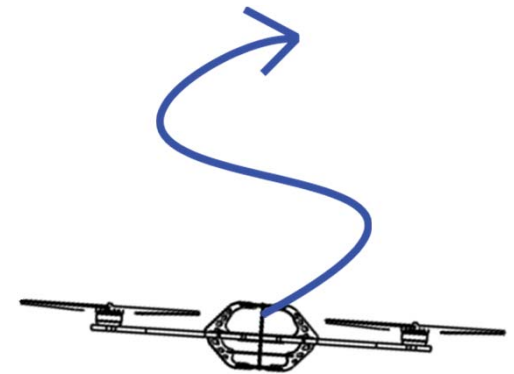
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## PREREQUISITES

- **Dynamics model of system**
  - (i) in analytical form or
  - (ii) in form of a numerical dynamics simulation
- **Desired output trajectory**  $y^*(t)$ ,  $t \in [0, t_f]$ , and corresponding nominal input trajectory  $u^*(t)$ .
  - $(u^*(t), y^*(t))$  must satisfy the model equations.

## RESULT

- Learned input
- Estimated disturbance vector



# A | LIFTED-DOMAIN REPRESENTATION

**Dynamics model** of the physical system:  $\dot{\check{x}}(t) = f(\check{x}(t), \check{u}(t)), \quad \check{y}(t) = \check{x}(t).$

Consider small **deviations from nominal trajectory**.

$$\tilde{u}(t) = \check{u}(t) - u^*(t), \quad \tilde{x}(t) = \check{x}(t) - x^*(t), \quad \tilde{y}(t) = \check{y}(t) - y^*(t)$$

**Linearize and discretize.** Linear, time-varying difference equation.

$$\tilde{x}(k+1) = A_D(k)\tilde{x}(k) + B_D(k)\tilde{u}(k), \quad \tilde{y}(k) = \tilde{x}(k), \quad k \in \{0, \dots, N\}.$$

**Static mapping.** Representing one trial.

$$\underbrace{\begin{bmatrix} \tilde{x}(0) \\ \tilde{x}(1) \\ \tilde{x}(2) \\ \vdots \\ \tilde{x}(N) \end{bmatrix}}_x = \underbrace{\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ B_D(0) & 0 & \cdots & 0 & 0 \\ \Phi_{(1,1)}B_D(0) & B_D(1) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \Phi_{(N-1,1)}B_D(0) & \Phi_{(N-1,2)}B_D(1) & \cdots & B_D(N) & 0 \end{bmatrix}}_F \underbrace{\begin{bmatrix} \tilde{u}(0) \\ \tilde{u}(1) \\ \tilde{u}(2) \\ \vdots \\ \tilde{u}(N) \end{bmatrix}}_u$$

With  $\Phi_{(l,m)} = A_D(l)A_D(l+1)\cdots A_D(m)$ ,  $l < m$ , and  $\tilde{x}(0) = 0$ .

# A | ITERATION-DOMAIN MODEL

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For each trial  $j$ ,  $j \in \{1, 2, \dots\}$ ,

$$y_j = F u_j + d_j + \mu_j.$$

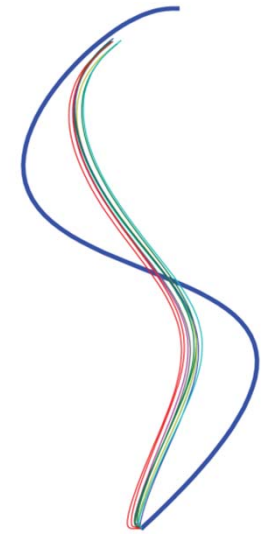
**Recurring disturbance**  $d_j$ .

Unknown. Only small changes between iterations:

$$d_j = d_{j-1} + \omega_{j-1}.$$

**Noise**  $\mu_j$ .

Unknown. Changing from iteration to iteration.

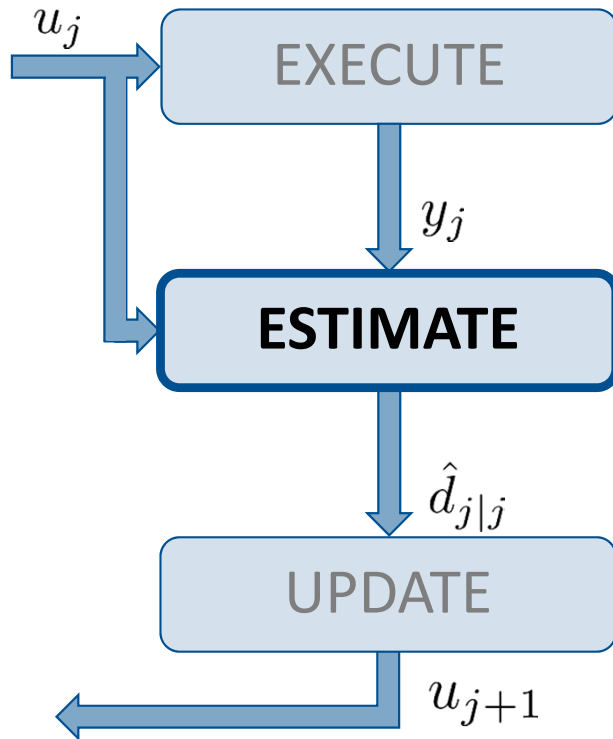


$\mu_j, \omega_j$  — trial-uncorrelated,  
zero-mean Gaussian  
noise

From trial to trial our knowledge about  $d_j$  improves.



# A | STEP 1: ESTIMATION



**UPDATE OF DISTURBANCE ESTIMATE**  
via **Kalman filter** in the iteration domain:

estimates the repetitive disturbance  $d_j$   
by taking into account all past measurements.

Prediction step:

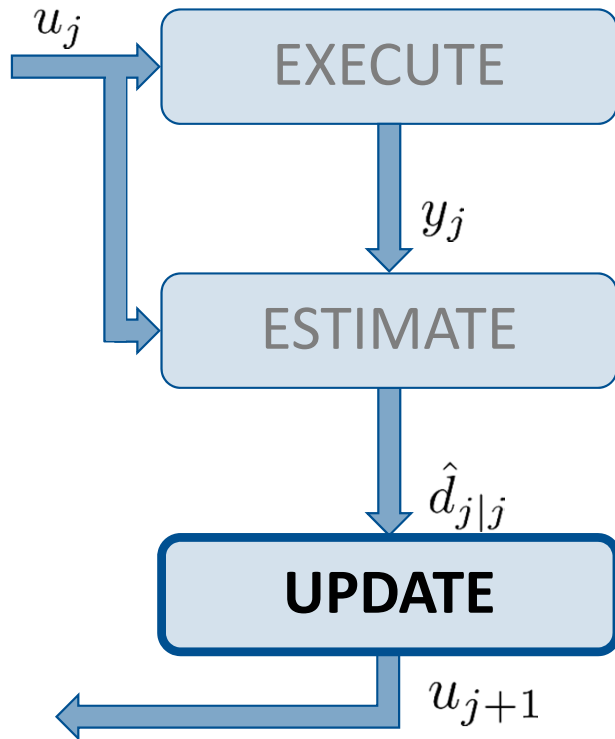
$$d_j = d_{j-1} + \omega_{j-1}.$$

Measurement update step:

$$y_j = F u_j + d_j + \mu_j.$$

➔ Obtain  $\hat{d}_{j|j}$ .

# A | STEP 2: UPDATE



**INPUT UPDATE** via **convex optimization**:

minimizes the tracking error in the next trial:

$$\mathbb{E}[y_{j+1} | \text{all past measurements}] = F u_{j+1} + \hat{d}_{j|j}.$$

$$\min_{u_{j+1}} \left\| F u_{j+1} + \hat{d}_{j|j} \right\|_p \quad p \in \{1, 2, \infty\}$$

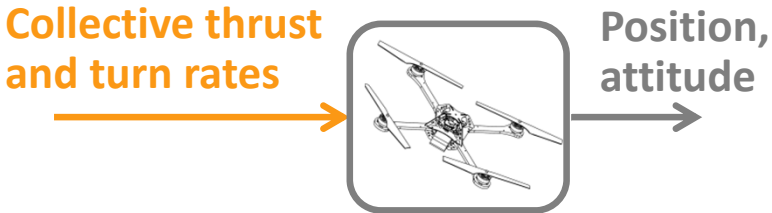
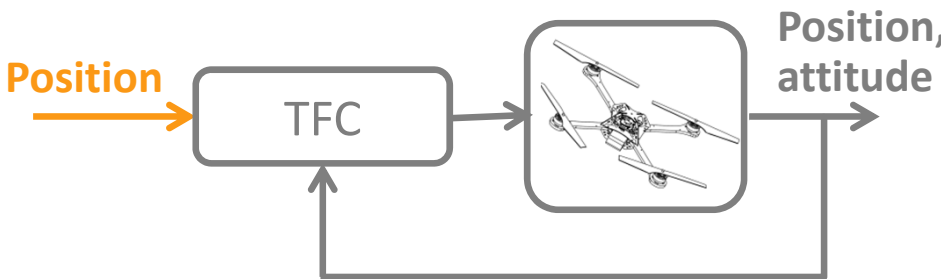
subject to

$$u_{\min} \leq u_{j+1} \leq u_{\max}$$

$$x_{\min} \leq x_{j+1} \leq x_{\max}$$

➔ Obtain  $u_{j+1}$ .

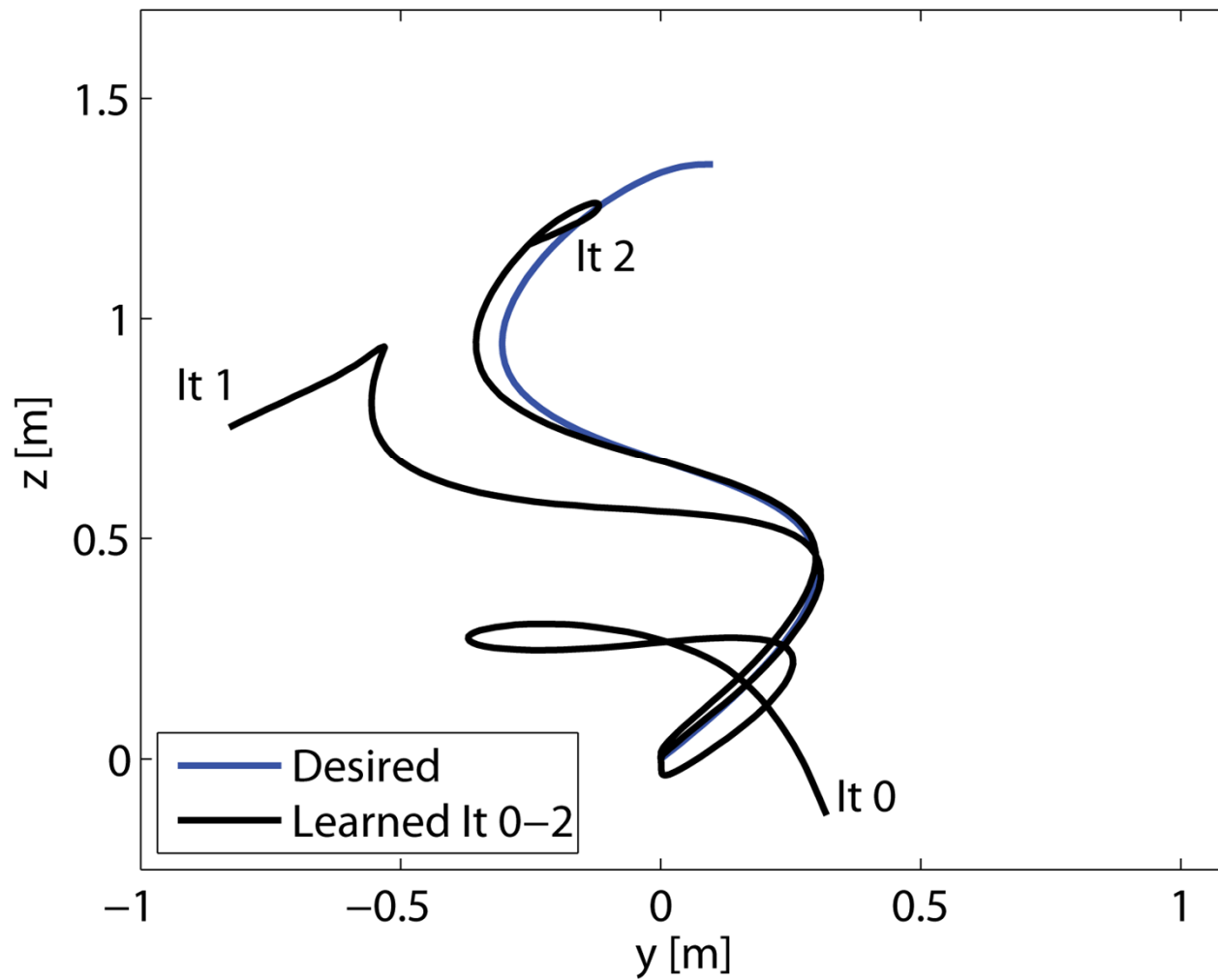
# A | TWO EXPERIMENTAL SCENARIOS

SCENARIO 1	SCENARIO 2
<ul style="list-style-type: none"> <li>No feedback from motion capture cameras during task execution</li> </ul>	<ul style="list-style-type: none"> <li>Camera information is used.</li> </ul>
 <p>The diagram shows a quadcopter drone in a rounded rectangular box. An orange arrow labeled 'Collective thrust and turn rates' points into the box from the left. A grey arrow labeled 'Position, attitude' points out of the box to the right.</p>	 <p>The diagram shows a quadcopter drone in a rounded rectangular box. An orange arrow labeled 'Position' points into a rounded rectangular block labeled 'TFC' from the left. A grey arrow points from the 'TFC' block to the drone. A feedback loop arrow points from the drone's output 'Position, attitude' back to the 'TFC' block.</p>
<ul style="list-style-type: none"> <li>Analytical model</li> </ul>	<ul style="list-style-type: none"> <li>Model via numerical simulation</li> </ul>
<ul style="list-style-type: none"> <li>2D quadcopter model</li> </ul>	<ul style="list-style-type: none"> <li>3D quadcopter model</li> </ul>
<ul style="list-style-type: none"> <li>Constraints on single motor thrusts and turn rates.</li> </ul>	

# A | SCENARIO 1: state trajectories

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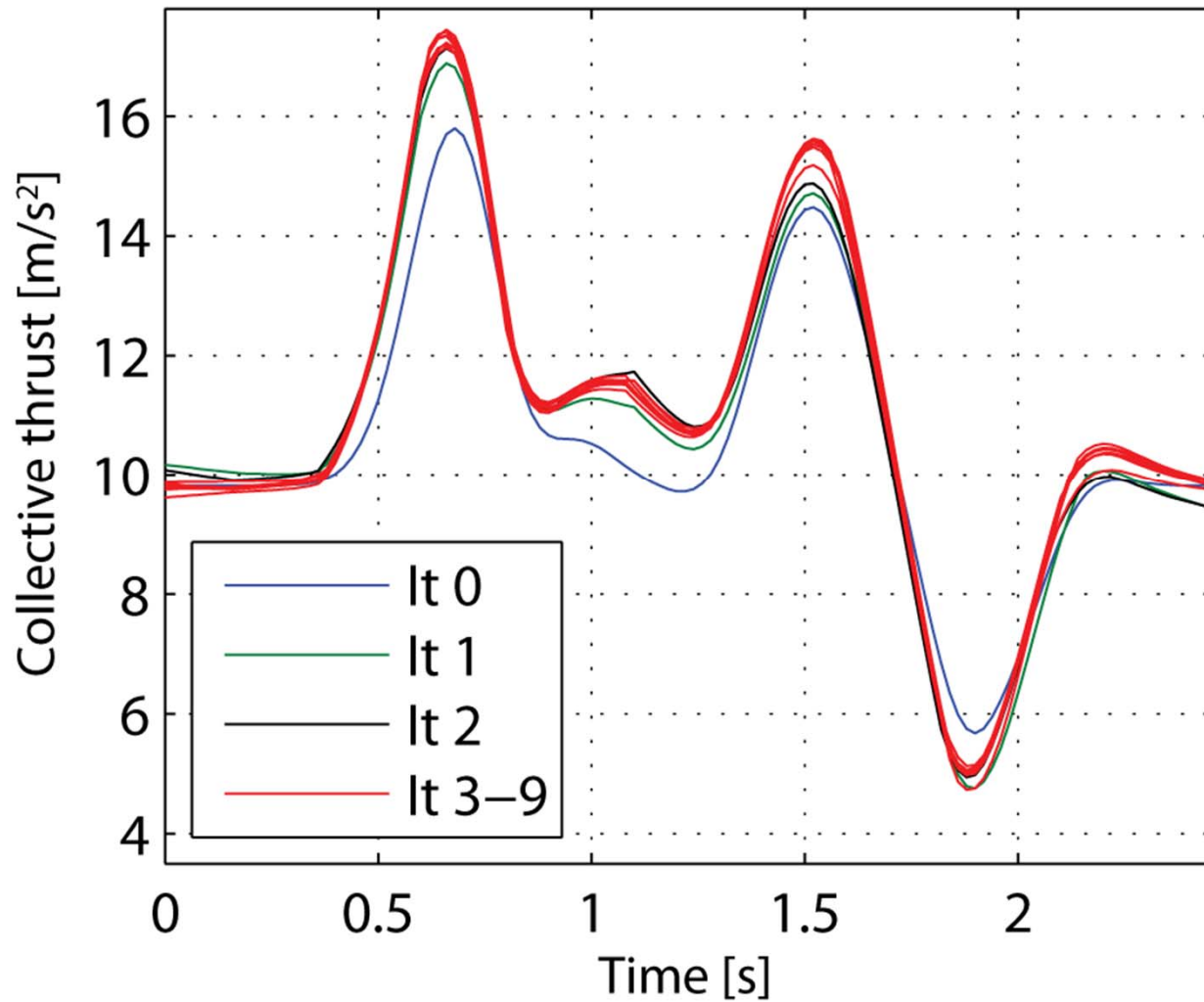
**S-shaped trajectory.**





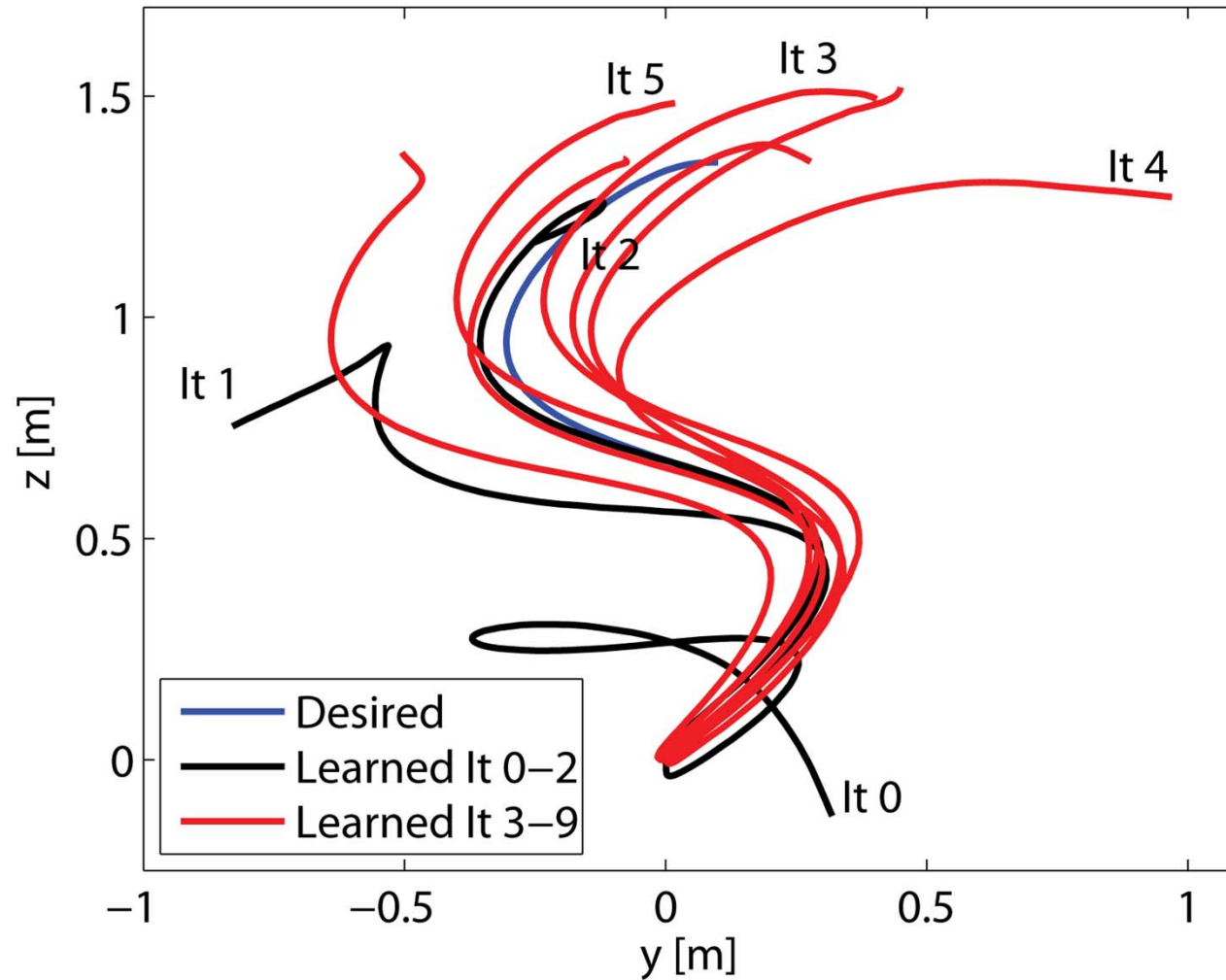
# A | SCENARIO 1: input trajectories

**S-shaped trajectory.**

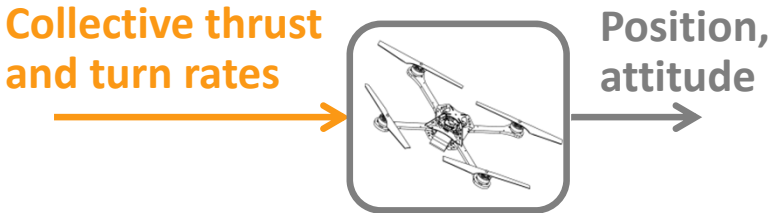
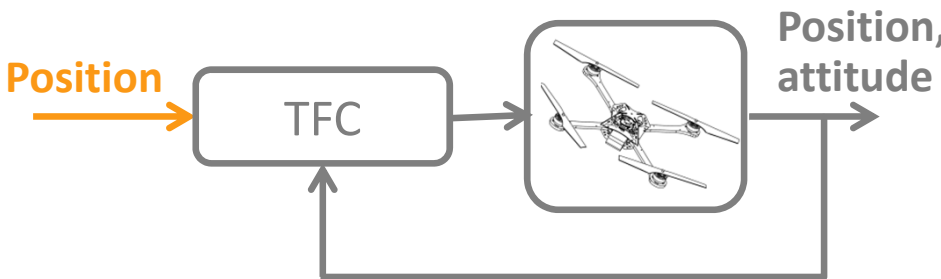


# A | SCENARIO 1: state trajectories

S-shaped trajectory.



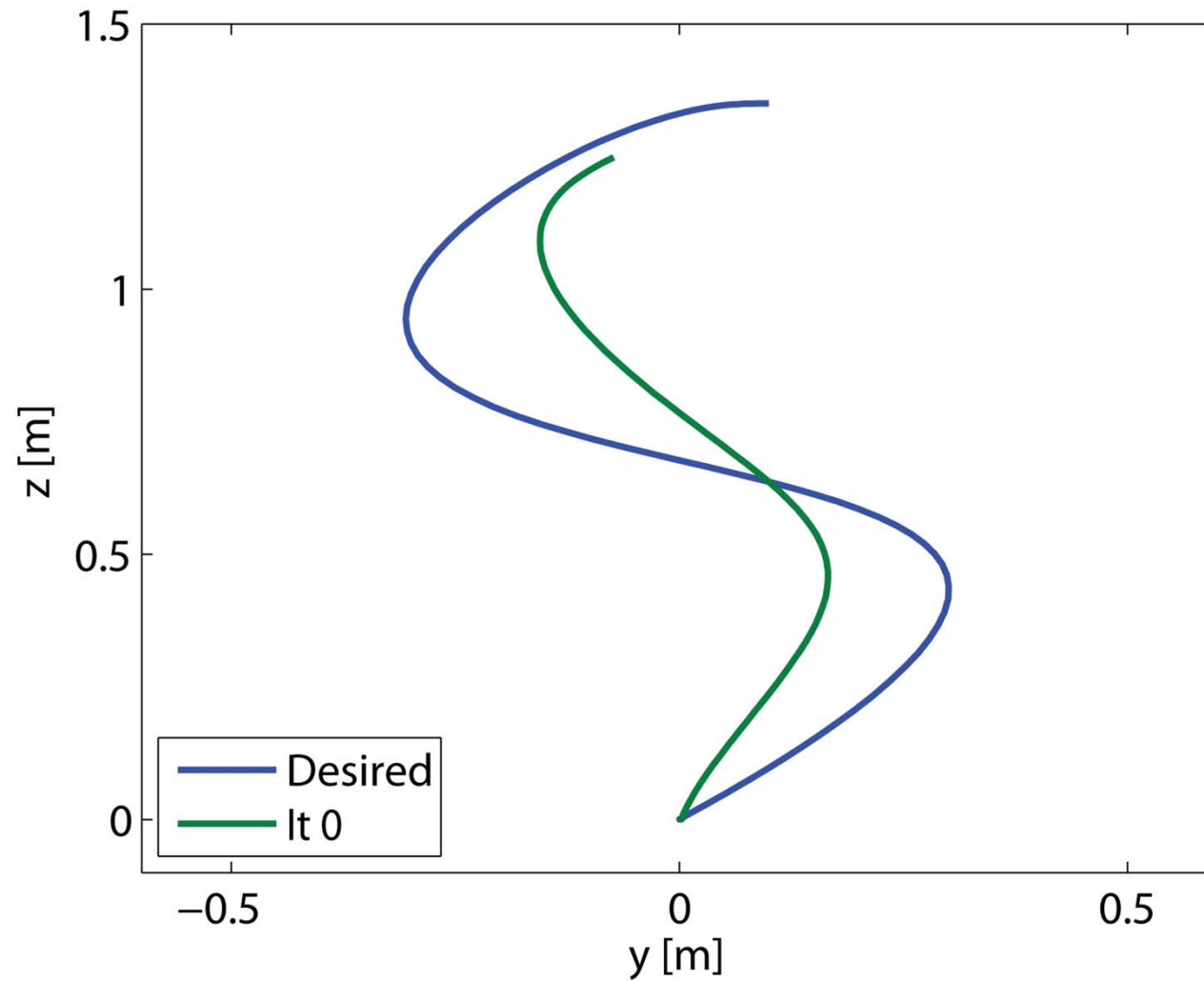
# A | TWO EXPERIMENTAL SCENARIOS

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<ul style="list-style-type: none"><li>Constraints on single motor thrusts and turn rates.</li></ul>	

# A | SCENARIO 2: state trajectories

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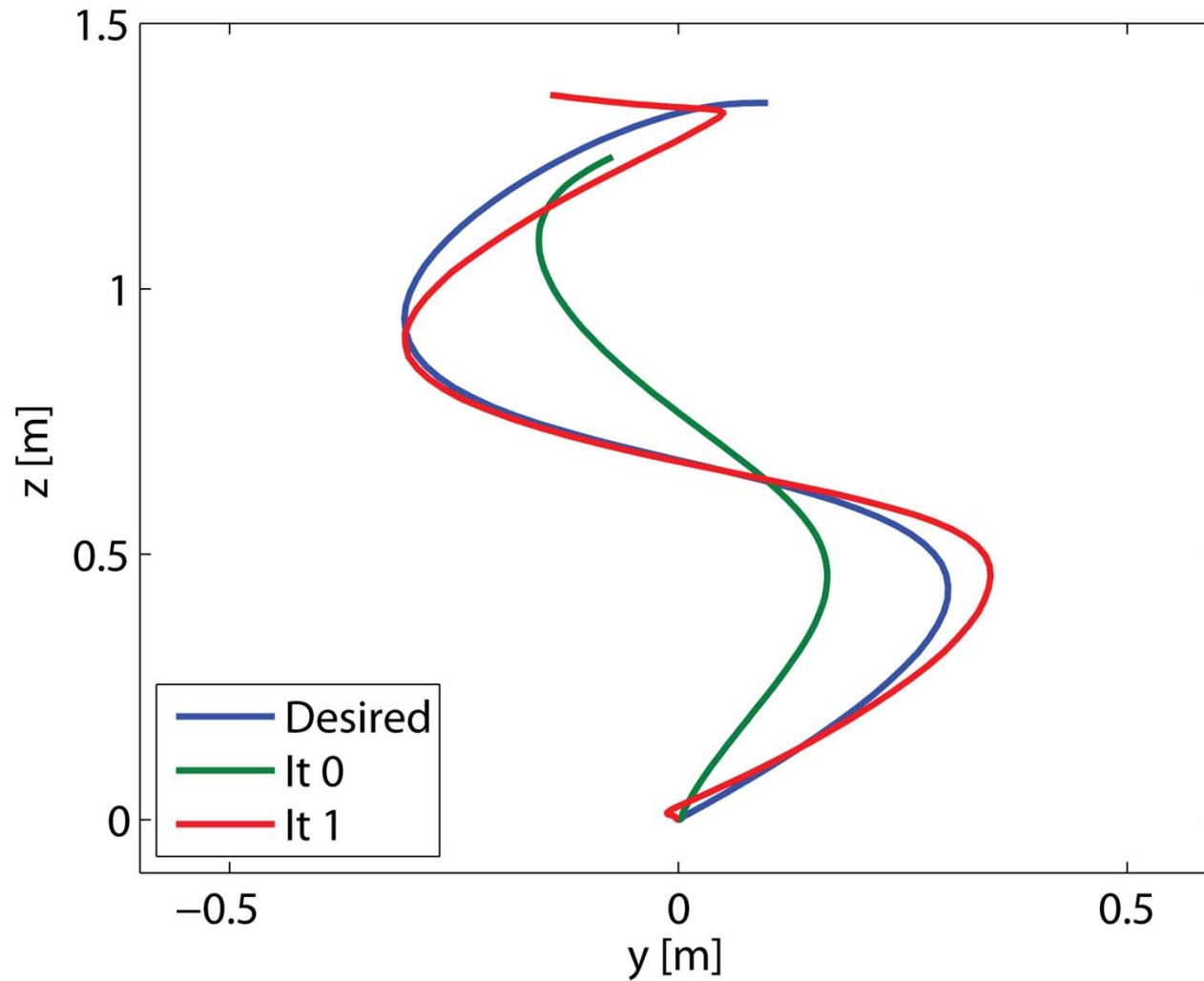
**S-shaped trajectory.**



# A | SCENARIO 2: state trajectories

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**S-shaped trajectory.**

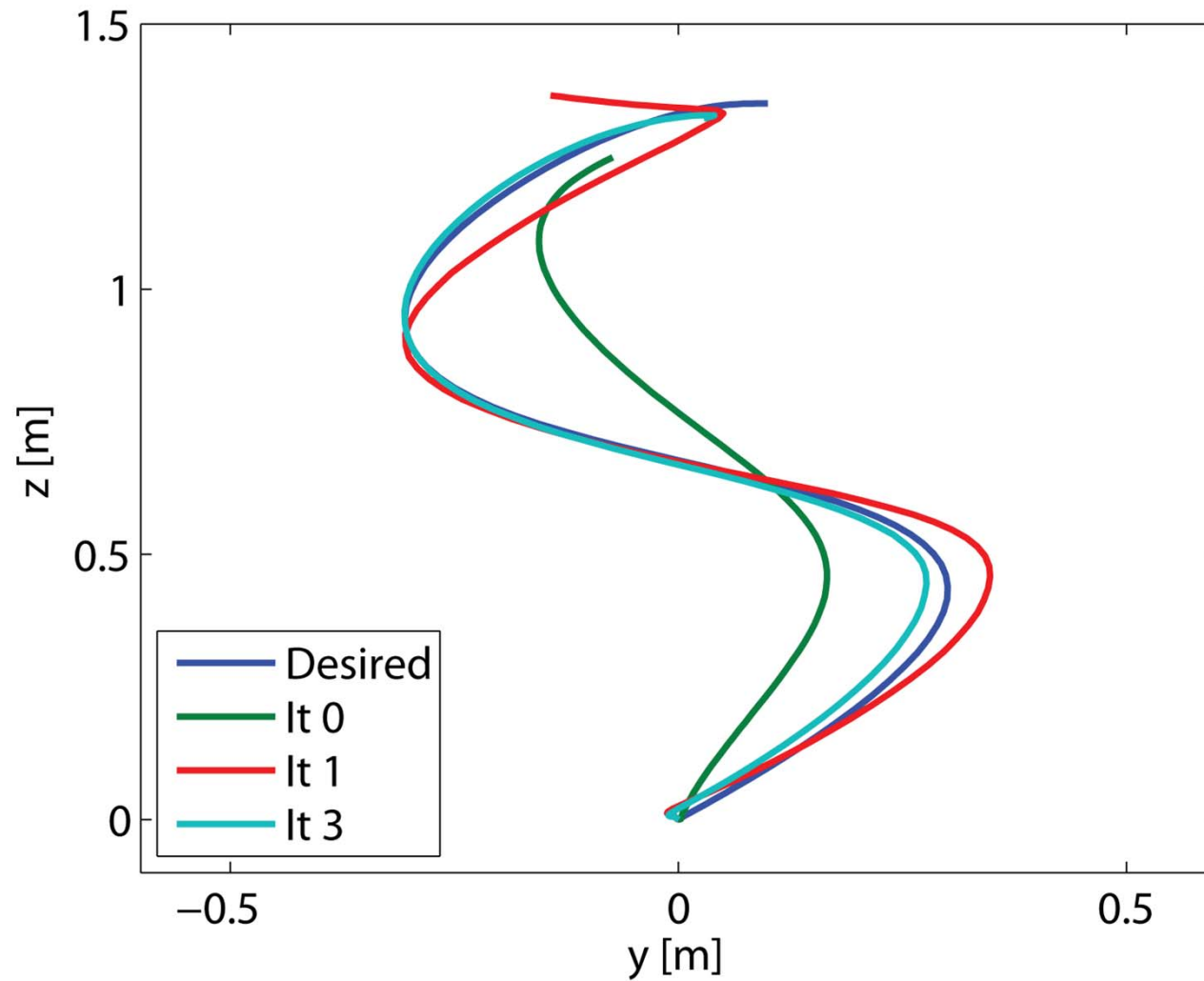




# A | SCENARIO 2: state trajectories

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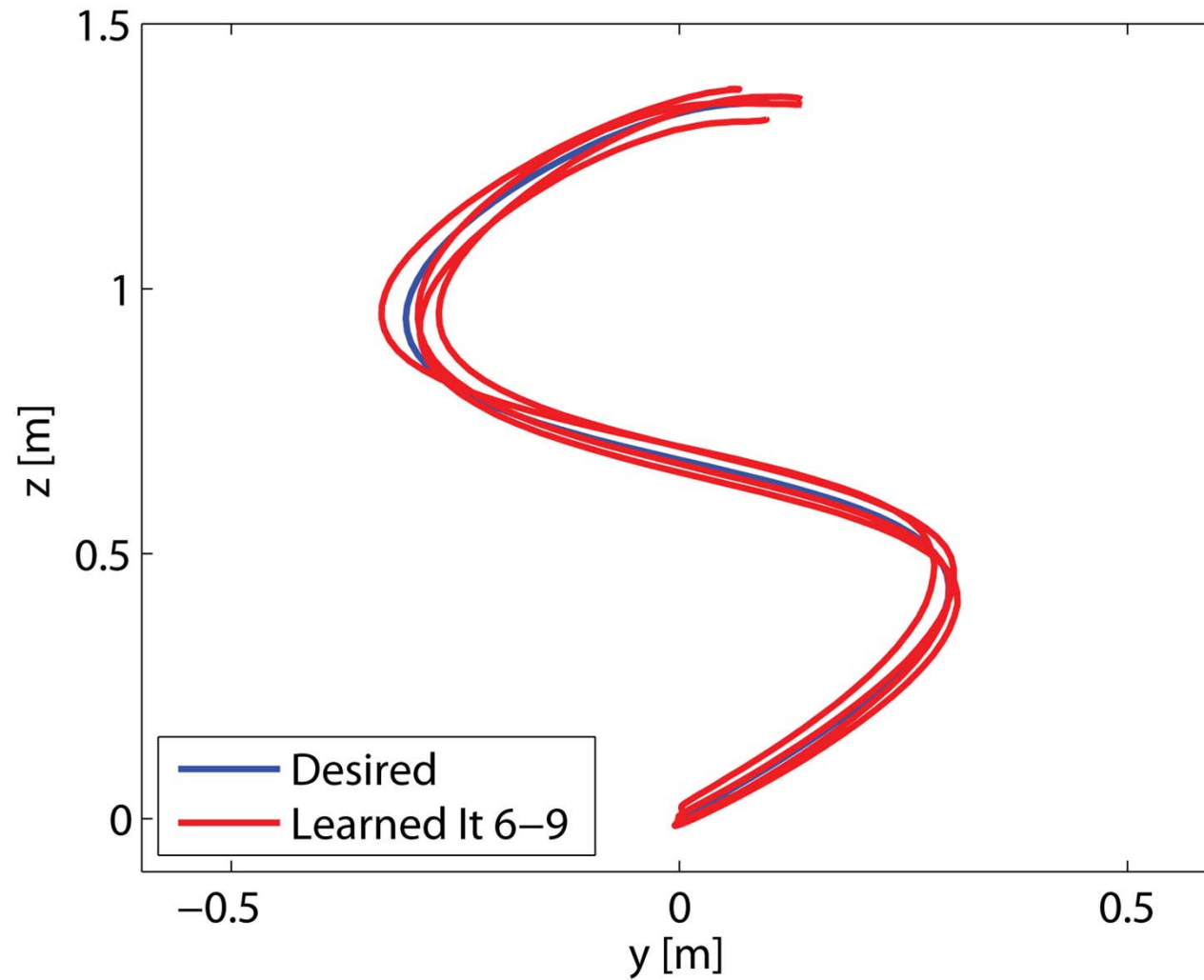
**S-shaped trajectory.**



# A | SCENARIO 2: state trajectories

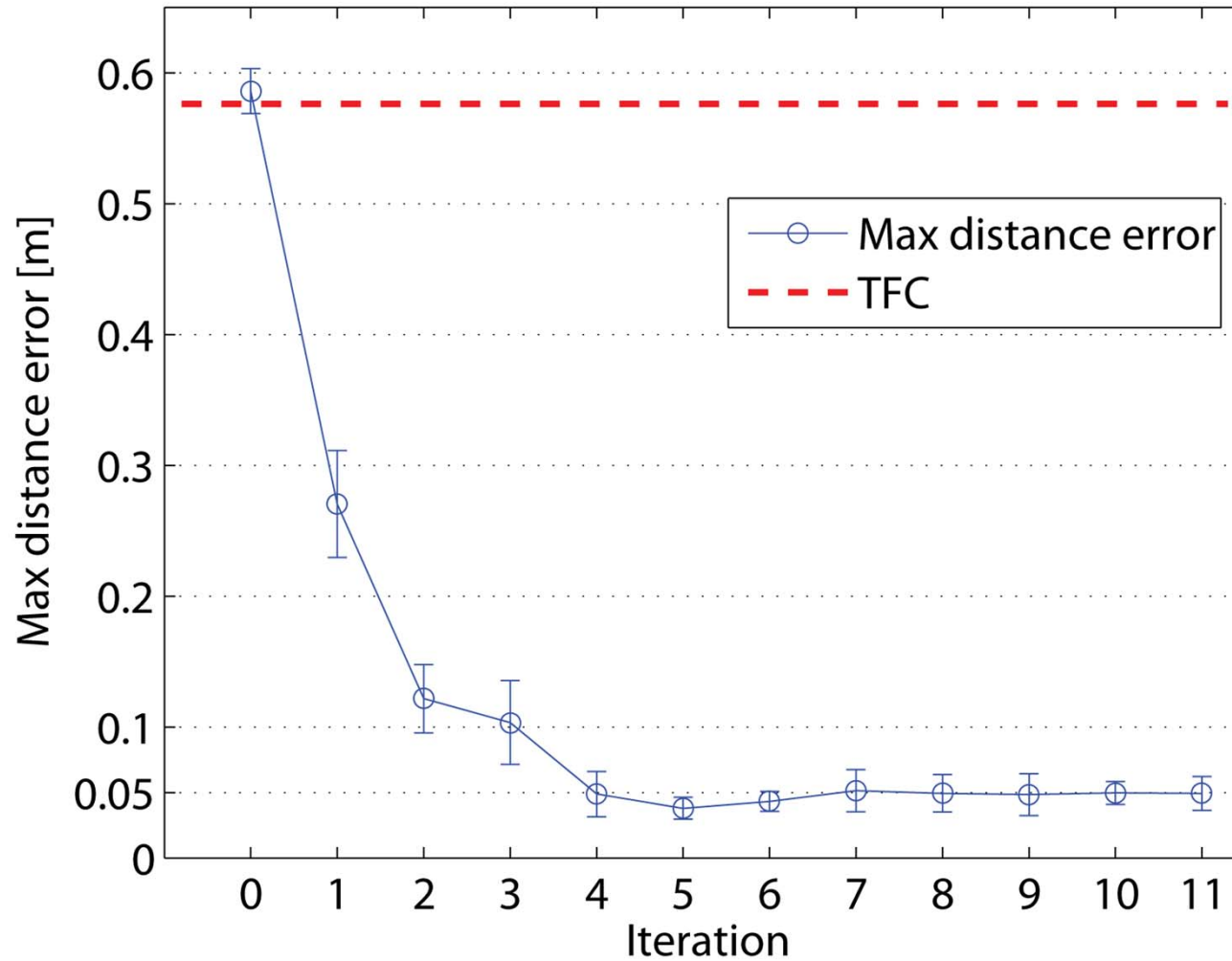
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**S-shaped trajectory.**



# A | SCENARIO 2: error convergence

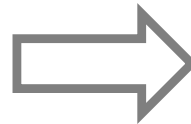
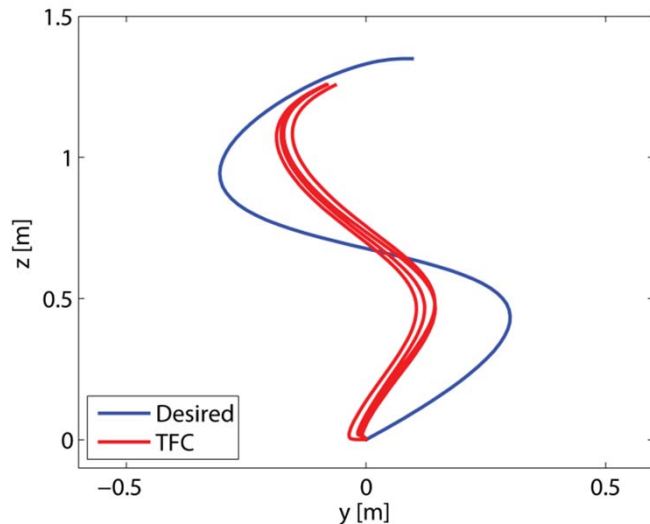
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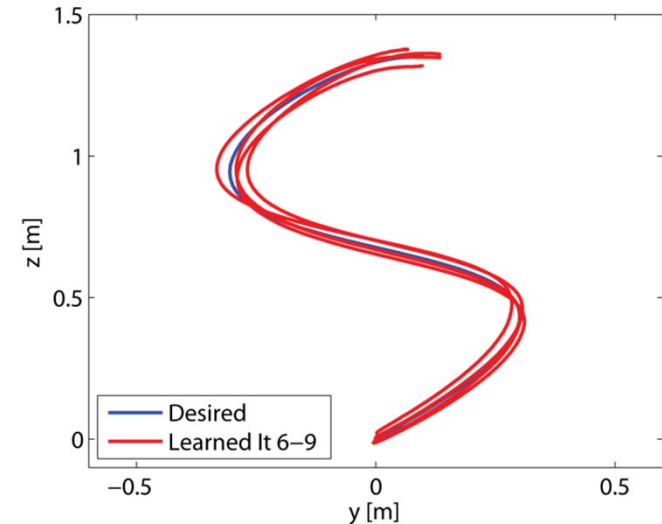
# A | SUMMARY

- **Prerequisites:** approximate model of system dynamics.
- **Efficient learning algorithm:** convergence in around 5-10 iterations.
- **Acausal compensation:** outperforms pure feedback control.

Scenario 2: without learning



with learning



**Powerful combination** Learning applied to feedback-control systems: compensation for repetitive and non-repetitive disturbances.

VIDEO: <http://tiny.cc/SlalomLearning>

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# Quadrocopter Slalom Learning



**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# OVERVIEW

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I. Introduction

II. Project A. Iterative learning for precise trajectory following

III. Project B. Learning of feed-forward parameters for rhythmic flight performances

a. Learning approach

b. Results

I. Summary



## B | PUBLICATIONS

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### *Peer-reviewed publications*

Schoellig, A. P., F. Augugliaro, and R. D'Andrea (2009):

“Synchronizing the motion of a quadrocopter to music.” In *Proceedings of IEEE International Conference on Robotics and Automation (ICRA)*.

Schoellig, A. P., F. Augugliaro, and R. D'Andrea (2010):

“A platform for dance performances with multiple quadrocopters.” In *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)–Workshop on Robots and Musical Expressions*.

Schoellig, A. P., M. Hehn, S. Lupashin, and R. D'Andrea (2011): “Feasibility of motion primitives for choreographed quadrocopter flight.” In *Proceedings of the American Control Conference (ACC)*.

Schoellig, A. P., C. Wiltsche, and R. D'Andrea (2012):

“Feed-forward parameter identification for precise periodic quadrocopter motions.” In *Proceedings of the American Control Conference (ACC)*.

**Joint work with Federico Augugliaro (Bachelor/Master student) and Clemens Wiltsche (semester project).**

VIDEO: <http://tiny.cc/DanceWith3>

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# Dancing Quadrocopters

*Rise Up*

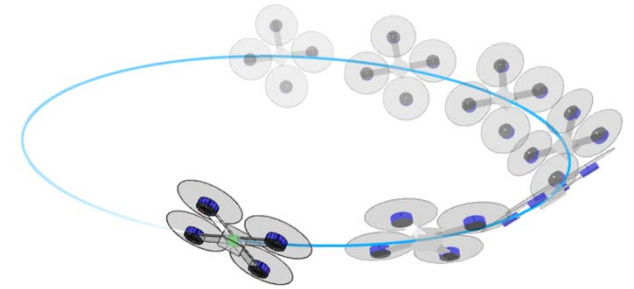


**ETH** zürich

## B | LEARNING APPROACH

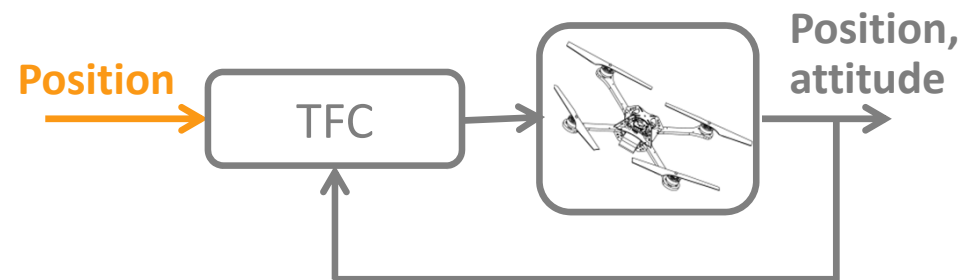
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**Task:** Precise tracking of *periodic* motions.



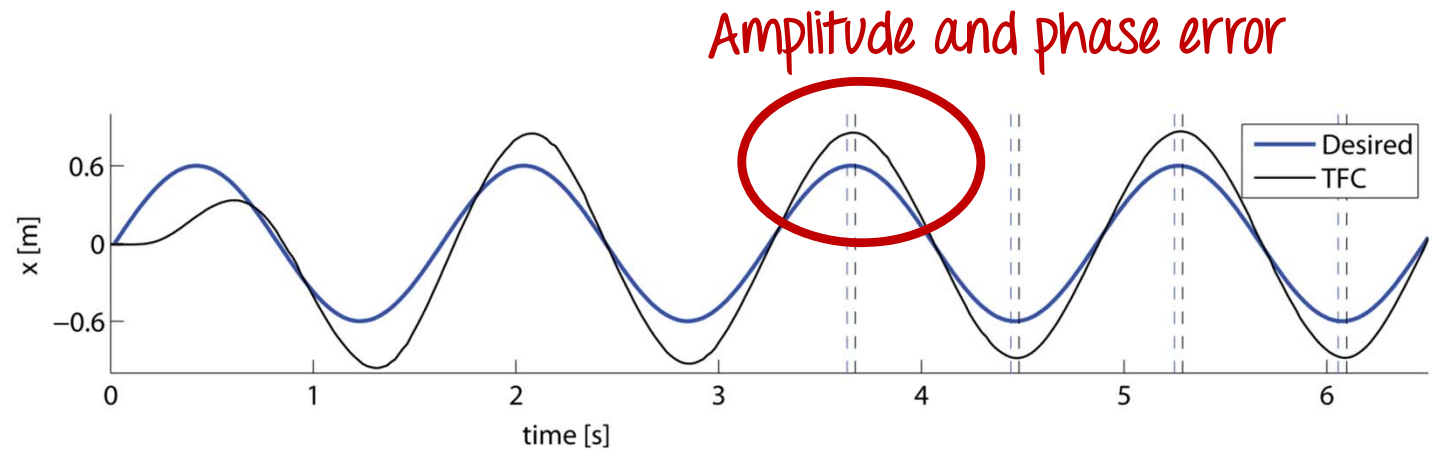
### Features:

- Learning through a dedicated identification routine performed prior to flight performance.
- Adaptation of only a few *input parameters*.

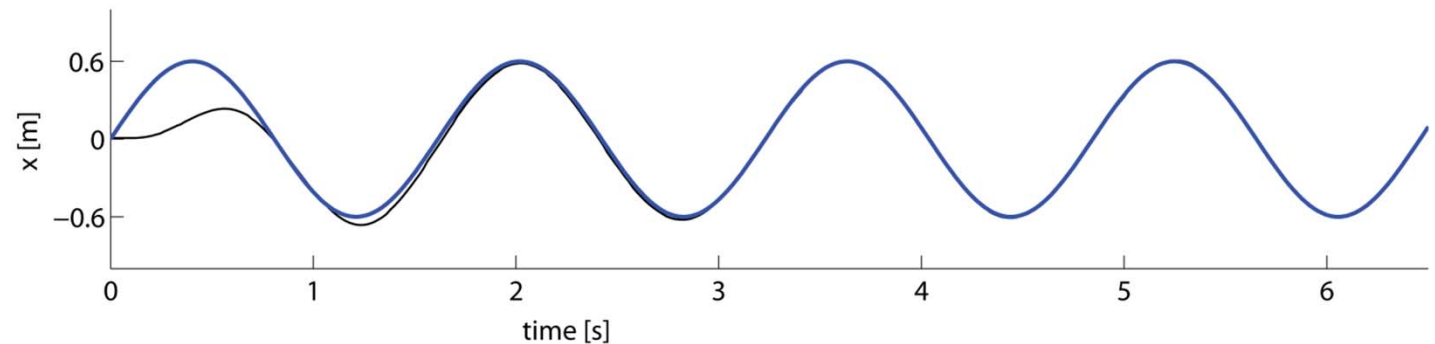


## B | LEARNING APPROACH

PURE FEEDBACK



WITH LEARNED  
CORRECTION  
FACTORS



For each directional motion component and frequency, we learn:

- (1) amplitude correction factor,
- (2) additive phase correction.

VIDEO: <http://tiny.cc/Armageddon>

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# Armageddon

@ the Flying Machine Arena

April 2011



**ETH**

Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

# OVERVIEW

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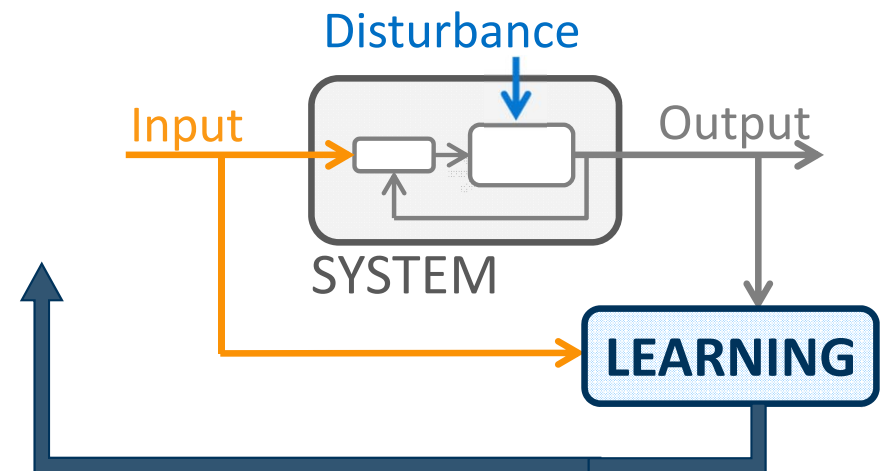
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# SUMMARY

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**Repetitive** error components can be effectively compensated for by learning from past data.

Result is an **improved tracking performance**.





# RESEARCH SUPPORT STAFF

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Igor Thommen



Carolina Flores



*Thank you!*

Hans Ulrich Honegger



Marc Corzillius



# IT FOLLOWS...

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Live demonstration  
in the Flying Machine Arena

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Doctoral examination – July 30, 2012

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