

Dynamic Programming and Optimal Control

Recitation Session

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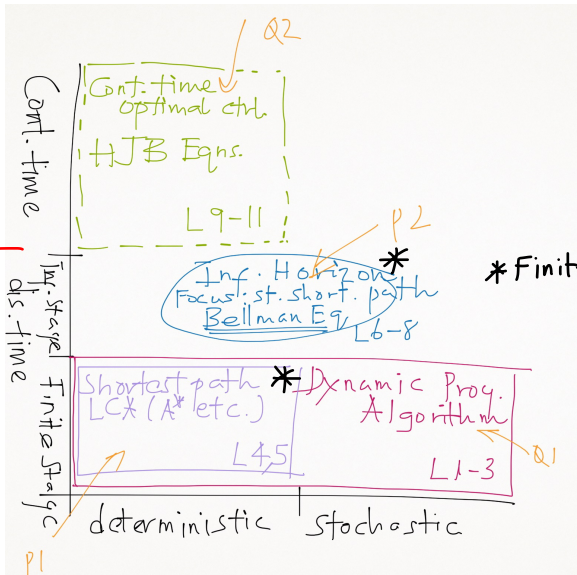
Plan for Today

1. The Dynamic Programming Algorithm (DPA)
2. Deterministic Systems and the Shortest Path (SP)
3. Infinite Horizon Problems, Stochastic SP
4. Deterministic Continuous-Time Optimal Control

Overview

Differential Eq.
 $\dot{x} = f(x, u)$

Difference Eq.
 $x_{k+1} = f_k(x_k, u_k)$



Dynamic Programming Algorithm

System dynamics:

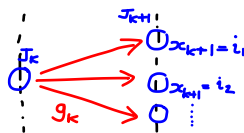
$$x_{k+1} = f_k(x_k, u_k, w_k), \quad k = 0, 1, \dots, N-1$$

time varying system

Cost:

$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k)$$

DPA:



$$J_N(x_N) = g_N(x_N),$$

$$J_k(x_k) = \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} \{g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k))\}$$

Ex 1: Stage k is not always a time step.

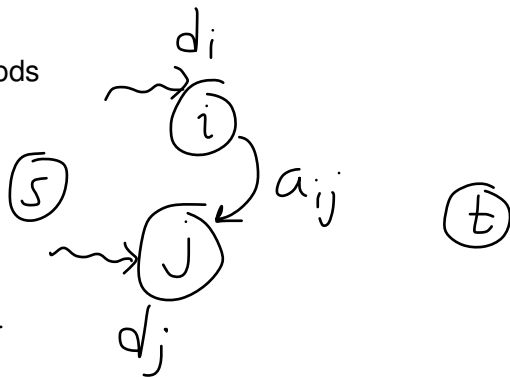
State Augmentation

x_k : summarizes past information that is relevant for future optimization.

Example #2

Shortest Path Problems

Label Correcting Methods



Ex 3: practice at least once!

Note: Know the difference between Label correcting variants:

Breadth first, Depth first
pg 84 (Bertsekas)

Shortest Path Problems

Viterbi Algorithm

Stochastic Shortest Path Problems

Definition

Stochastic Shortest Path Problems

Policy Iteration, Value Iteration, ~~Linear Programming~~

Continuous time optimal control

System dynamics: $\dot{x}(t) = f(x(t), u(t))$, $0 \leq t \leq T$, $x(0) = x_0$

Cost: $h(x(T)) + \int_0^T g(x(t), u(t)) dt$

HJB:

$$0 = \min_{u \in U} \left[g(x, u) + \frac{\partial V(t, x)}{\partial t} + \left(\frac{\partial V(t, x)}{\partial x} \right)^T f(x, u) \right], \quad \forall x, t \in [0, T],$$

$$V(T, x) = h(x), \quad \forall x.$$

DPA:

$$J_N(x_N) = g_N(x_N),$$

$$J_k(x_k) = \min_{u_k \in U_k(x_k)} \mathbb{E}_{w_k} \{ g_k(x_k, u_k, w_k) + J_{k+1}(f_k(x_k, u_k, w_k)) \}$$

Continuous time optimal control

Minimum principle:

$$H(x, u, p) = g(x, u) + p^T f(x, u)$$

$$\dot{x}^*(t) = \frac{\partial H(x^*(t), u^*(t), p(t))}{\partial p}, \quad x^*(0) = x_0,$$

$$\dot{p}(t) = -\frac{\partial H(x^*(t), u^*(t), p(t))}{\partial x}, \quad p(T) = \frac{\partial h(x^*(T))}{\partial x}$$

$$u^*(t) = \operatorname{argmin}_{u \in U} H(x^*(t), u(t), p(t))$$